

Shifted convolution sums and subconvexity bounds for automorphic L -functions

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Abstract

Let $f(z) = \sum a(m)m^{(k-1)/2}e(mz) \in S_k(N, \chi_1)$ be a primitive cusp form of weight k , level N and character χ_1 . For a smooth weight function g with support in $[M_1, 2M_1] \times [M_2, 2M_2]$ and positive integers l_1, l_2, h the bound

$$\sum_{l_1 m_1 - l_2 m_2 = h} a(m_1) \overline{a(m_2)} g(m_1, m_2) \ll_{\varepsilon} (l_1 M_1 + l_2 M_2)^{1/2 + \theta + \varepsilon}$$

with $\theta = \frac{7}{64}$ is shown. As applications the shifted sum $\sum_{m \leq M} a(m) \overline{a(m+h)}$ is bounded non-trivially for $h \ll M^{64/39 - \varepsilon}$. Furthermore, the subconvexity bound

$$L_f(1/2 + it, \chi) \ll_{\varepsilon} D^{71/167 + \varepsilon}$$

for the L -function attached to the twist of f with a primitive character χ to modulus D is obtained.

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