Minisymposium 7 - Stochastic algorithms and Markov processes

Local Spectral Gaps on the Mean Field Ising Model and Multilevel MCMC methods

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I consider the Metropolis Markov Chain based on the nearest neighbor random walk on the positive half of the Mean Field Ising Model, i.e., on those vectors from $\{-1,1\}^N$ which contain more 1 than -1. Using randomly-chosen paths I prove a lower bound for the Spectral Gap of this chain which is of order N^{-2} and which does not depend on the inverse temperature β.

In conjunction with decomposition results such as those in Jerrum, Son, Tetali and Vigoda (2004) this result may be useful for bounding the spectral gaps of more complex Markov chains on the Mean Field Ising Model which may be decomposed into Metropolis chains. As an example, I apply the result to two Multilevel Markov Chain Monte Carlo algorithms, Swapping and Simulated Tempering. Improving a result by Madras and Zheng (2002), I show that the spectral gaps of both algorithms on the (full) Mean Field Ising Model are bounded below by the reciprocal of a polynomial in the lattice size N and in the inverse temperature β.