Minisymposium 16 - Set Theory

Definable well–orders of $H(\omega_2)$ and forcing axioms

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This talk deals with the problem of building set-forcing extensions in which there is a simple definition, over the structure $\langle H(\omega_2), \in\rangle$ and without parameters, of a prescribed member of $H(\omega_2)$ or of a well–order of $H(\omega_2)$, possibly together with some strong forcing axiom.
I will present two theorems. The first one is an optimal result, with respect to the logical complexity of the definitions involved, at the level of the structure $\langle H(\omega_2), \in, NS_{\omega_1}\rangle$. This result is a particular case of a much more general theorem applying to $H(\kappa^+)$ for every uncountable regular cardinal κ.
The second theorem I will present says that, under the assumption that there is a supercompact cardinal, there is a partial order forcing both the existence of a well–order of $H(\omega_2)$ definable, over $\langle H(\omega_2), \in\rangle$, by a formula without parameters, and that the forcing axiom PFA^{++} holds.