Minisymposium 24 - Probability and Geometry

Gradient estimates for positive harmonic functions, Harnack inequalities and heat kernel estimates on Riemannian manifolds, by stochastic analysis

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The talk is divided into three parts; we report on recent work with Bruce Driver, Anton Thalmaier and Feng-Yu Wang.
In the first part we prove gradient estimates for positive harmonic functions on Riemannian manifolds by using a Bismut type inequality which is derived by an integration by parts argument from an underlying submartingale. A crucial but elementary ingredient is that positive local martingales have moments of order $\beta \in]0,1[$ dominated by $C_\beta\, z$ where C_β is a universal positive constant and z is the starting point of the local martingale.
In the second part, coupling by parallel translation, along with Girsanov's theorem, is used to establish a new version of a dimension-free Harnack inequality for diffusion semigroups on Riemannian manifolds with Ricci curvarture unbounded below. As an application, in the symmetric case, a Li-Yau type heat kernel bound is presented for such semigroups.
In the third part we prove Li-Yau and Hamilton estimates for heat kernels in compact manifolds by replacing classical maximum principle by submartingale arguments. For Hamilton's estimate, we demonstrate that a certain quadratic form valued semimartingale can not exit the set of nonpositive quadratic forms, outside of which it would have a drift contradicting its known asymptotic behaviour.